METRIK GRAFLARDA HILFER OPERATORI QATNASHGAN VAQT BO‘YICHA KASR TARTIBLI DIFFERENSIAL TENGLAMALAR UCHUN TESKARI MASALA

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ANNOTATSIYA. *Bugungi kunda metrik graflarda Hilfer operatori qatnashgan vaqt bo’yicha kasr tartibli differensial tenglamalar uchun teskari masala yechish matematika fanining barcha darajalarida dolzarblik kasb etmoqda. Negaki, tarmoqlangan sohalarda to‘lqin tarqalishi va diffuziya jarayonlari ko‘plab sohalarda uchraydi. Masalan, neyron birikmalarida nerv impulslarining tarqalishi, arterial qon aylanish sistemasida impuls tarqalishi, elektr tarmoqlaridagi to‘lqin jarayonlarini keltirish mumkin. Bunday jarayonlar odatda metrik graflarda differensial tenglamalar uchun boshlang‘ich-chegaraviy masalalar yordamida modellashtiriladi.*

Metrik graflarda differensial tenglamalar sohasini o‘rganish nisbatan yangi soha hisoblanib, asosan 1990-yillarga kelib grafda chiziqli Shredinger tenglamasi o‘rganila boshlangan. Keyinchalik esa graflarda boshqa turdagi tenglamalar 2010-yillarga kelib o‘rganila boshlangan. Bulardan chiziqsiz Shredinger tenglamasi, Sin-Gardon tenglamasi, KdV tenglamalari bo‘yicha olingan natijalarni keltirishimiz mumkin.

Ushbu maqolada Hilfer operatorining ba'zi xossalarini o'rganishga hamda bu xossalarni kasr tartibli differensial tenglamalar uchun teskari masalalarni metrik graflarda yechishning nazariy va amaliy jihatlari yoritiladi.

KALIT SO'ZLAR. *Hilfer operatori, chegaraviy masala, teskari masala, kasr tartibli hosila, differensial tenglama, integral, silliq funksiyalar.*

KIRISH. Matematik analizning ixtiyoriy kasr tartibli hosila va integrallarni o'rganish va qo'llashga bag'ishlangan sohasi funksiyalar nazariyasi, integral va differensial tenglamalar bilan bog'liq bo'lgan ko'p yillik tarixga ega. Hozirgi fan taraqqiyotida amaliy jarayonlarni matematik modelini tuzishda va xususiy hosilali giperbolik va aralash tipdagi tenglamalar uchun qo'yilgan chegaraviy masalalarni yechishda kasr tartibli integro-differensial operatorlar muhim o'rin tutadi.

Kasr tartibli differensiallash va integrallash tushunchasi odatda Riman-Liuvill, Kaputo, Hilfer nomlari bilan bog'lanadi. Liuvill, Riman, Letnikov, Veyl, Adamar, Hilfer va boshqa mashhur matematiklar hozirgi vaqtda matematik analiz, differensial tenglamalar va matematik fizika tenglamalari kabi bir necha fanlarning butun bir yo'nalishiga aylanib borayotgan kasr tartibli integro-differensial operatorlarning rivojlanishiga katta hissa qo'shganlar. Riman-Liuvill, Kaputo va Hilfer kasr tartibli funksiyalarining differensial tenglamalar uchun boshlang'ich shart masalalari oralig'ida hisoblanadi. Mazkur ishda metrik graflarda Hilfer operatori qatnashgan vaqt bo'yicha kasr tartibli differensial tenglamalar uchun nolokal va Kaputa kasr-tartibli operatori qatnashgan tenglama uchun metrik graflarda teskari masalalar bir qiymatli yechib ko'rsatilgan.

Tarmoqlangan sohalarda to'lqin tarqalishi va diffuziya jarayonlari ko'plab sohalarda uchraydi. Misol sifatida neyron birikmalarida nerv impulslarining tarqalishi, orterial qon aylanish sistemasida puls tarqalishi, elektr tarmoqlaridagi to'lqinlar tarmoqlangan sohalarda issiqlik jarayonlarini keltirish mumkin. Bunday jarayonlar odatda metrik graflarda differensal tenglamalar uchun chegaraviy masalalar yordamida modellashtiriladi.

ADABIYOTLAR TAHLILI VA METODOLOGIYA. Ushbu mavzuni tadqiq qilishda xorijlik mutahasislar G. Berkolaiko va P.Kuchment uzluksiz funksiyalarning standart fazosini aniqlashda foydalangan bo’lsa [1], J. Friedman va Jean-Pierre Tillichlar metrik grafni har bir uchidagi shartlarni yozishib, bir jinsli chiziqli sistema olishgan hamda λ ga bog‘liq matritsasi xos qiymatlarda yagona bo‘lishini isbotlashgan.[2]

NATIJALAR. Teskari masala. $B_k \times (0,T)$ sohada

$$ {}_C D_{0t}^{\alpha} u^{(k)}(x,t) = a u_{xx}^{(k)}(x,t) + b\, {}_C D_{0t}^{\alpha} u_{xx}^{(k)}(x,t) + $$

$$ + c u^{(k)}(x,t) + f^{(k)}(x,t) + h^{(k)}(x,t), $$

tenglamani hamda

$$ u^{(1)}(0,t) = u^{(2)}(0,t) = \ldots = u^{(j)}(0,t), \quad t \in [0,T], $$

$$ u_x^{(1)}(0,t) + u_x^{(2)}(0,t) + \ldots + u_x^{(j)}(0,t) = 0, \quad t \in [0,T], $$

hamda, chegaraviy

$$ u_x^{(k)}(l_k,t) = 0,\ t \in [0,T],\ k = \overline{1,j}, $$

va boshlang‘ich

$$ u^{(k)}(x,0) = \varphi^{(k)}(x),\ x \in B_k,\ k = \overline{1,j} $$

shartlarni va quyidagi

$$ u^{(k)}(x,T) = \psi^{(k)}(x),\ x \in B_k,\ k = \overline{1,j}. $$

shartlarni qanoatlantiradigan $\left(u^{(k)}(x,t), h^{(k)}(x)\right)$ funksiyalar juftligini toping.

Bu yerda $\psi^{(k)}(x)$ berilgan yetarli silliq funksiyalar, bundan tashqari

$$ \psi^{(1)}(0) = \psi^{(2)}(0) = .. = \psi^{(k)}(0), \psi_x^{(1)}(0) + \psi_x^{(2)}(0) + \ldots + \psi_x^{(j)}(0) = 0. $$

Shartlarni qanoatlantiradi.

$(u^{(k)}(x,t), h^{(k)}(x,t))$ masalaning umumlashtirilgan yechimlari funksiyalar juftligidir. Bu yerda

$$ u^{(k)}(x,t) \in C([0,l_k] \times [0,T]),\ u_x^{(k)}(x,t) \in C\left([0,l_k] \times (0,T)\right), $$

$$ {}_C D_{0t}^{\alpha} u^{(k)}(x,t), {}_C D_{0t}^{\alpha} u_{xx}^{(k)}(x,t) \in C\left((0,l_k)\times(0,T)\right),. $$

Teorema. Agar $f^{(k)}(x,t)$ va $\varphi^{(k)}(x)$, $\psi^{(k)}(x)$ funksiyalar uchun quyida berilgan

(1) $f^{(k)}(x,t) \in C\left([0,l_k]\times(0,T)\right)$ va $\dfrac{\partial}{\partial x} f^{(k)}(x,t)$ funksiyalar $B_k \times (0,T)$ da absolut integrallanuvchi

(2) $\varphi^{(k)}(x), \psi^{(k)}(x) \in C^2[0,l_k]$, $\dfrac{d^3}{dx^3}\varphi^{(k)}(x)$, $\dfrac{d^3}{dx^3}\psi^{(k)}(x)$ funksiyalar $(0,l_k)$ da absolyut integrallanuvchi

(3) $\varphi_x^{(1)}(l_1) + \varphi_x^{(2)}(l_2) + \ldots + \varphi_x^{(j)}(l_j) = 0, \quad \varphi_{xx}^{(1)}(0) + \varphi_{xx}^{(2)}(0) + \ldots + \varphi_{xx}^{(j)}(0) = 0,$

(4) $\psi_x^{(1)}(l_1) + \psi_x^{(2)}(l_2) + \ldots + \psi_x^{(j)}(l_j) = 0, \quad \psi_{xx}^{(1)}(0) + \psi_{xx}^{(2)}(0) + \ldots + \psi_{xx}^{(j)}(0) = 0$

shartlar bajarilsa, u holda masalaning yagona yechimi mavjud bo‘ladi va quyidagi ko’rinishda topiladi.

$$ u^{(k)}(x,t) = \sum_{n=0}^{\infty} \frac{1}{1+\beta\lambda_n^2}\Big[\frac{(1+\beta\lambda_n^2)[\psi_n - \varphi_n E_\alpha\left(-\mu_n^2 T^\alpha\right)] - F_n(T)}{G(T)} G(t) + $$

$$ + F_n(t) + \varphi_n E_\alpha\left(-\mu_n^2 t^\alpha\right)\Big] X_n^{(k)}(x), $$

$$ h^{(k)}(x,t) = \sum_{n=0}^{\infty} \Big[\frac{(1+\beta\lambda_n^2)[\psi_n - \varphi_n E_\alpha\left(-\mu_n^2 T^\alpha\right)] - F(T)}{G(T)}\Big] g(t) X_n^{(k)}(x). $$

MUHOKAMA. Metrik graflarda Hilfer operatori qatnashgan vaqt bo’yicha kasr tartibli differensial tenglamalar to’g’ri masala ko’rilib isbotlangan. Bu maqolada esa metrik graflarda Hilfer operatori qatnashgan vaqt bo’yicha kasr tartibli differensial tenglamalar uchun teskari masala ko’rilgan, buning yechimi mavjud va yoganaligi ko’rsatilgan. Bundan tarmoqlangan sohalarda to‘lqin tarqalishi va diffuziya jarayonlari ko‘plab sohalarda uchraydi. Misol sifatida neyron birikmalarida nerv impulslarining tarqalishi, orterial qon aylanish sistemasida puls tarqalishi, elektr tarmoqlaridagi to‘lqinlar tarmoqlangan sohalarda issiqlik jarayonlarini keltirish mumkin. Bunday jarayonlar odatda metrik graflarda differensal tenglamalar uchun chegaraviy masalalar yordamida modellashtiriladi.

XULOSA. “Metrik graflarda Hilfer operatori qatnashgan vaqt bo‘yicha kasr tartibli differensial tenglamalar uchun teskari masala” nomli ushbu ishda kasr tartibli parabolik tipdagi tenglamalar uchun chegaraviy masalalarni yechish o‘rganildi. Qo‘yilgan masalalarning bir qiymatli yechildi.

Kaputa kasr-tartibli operatori qatnashgan tenglama uchun metrik graflarda teskari masala qaraldi va masala yechimining yagonaligi va mavjudligi yoritildi.

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