SOME APPLICATIONS OF THE DERIVATIVE OF A FUNCTION

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Annotation. This article describes the concept of a derivative of a function and its practical application. Examples of product application are provided. The mechanical description of the derivative is also considered. It is known that the concept of a derivative of a function is one of the first basic concepts of mathematical analysis, and problems leading to the concept of a derivative include such problems as checking the rectilinear motion of a rigid body, the motion of an object thrown vertically upward, or the motion of a piston in an engine cylinder . When considering such movements, we ignore the specific size and shape of the body and imagine it as a moving material point.

Key words: Addition of a function, addition of an argument, finite limit, derivative, differential.

НЕКОТОРЫЕ ПРИМЕНЕНИЯ ПРОИЗВОДНОЙ ФУНКЦИИ

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Аннотация. В данной статье описывается понятие производной функции и его практическое применение. Приведены примеры применения производной. Также рассматривается механическое описание производной. Известно, что понятие производной функции является одним из первых основных понятий математического анализа, а к числу задач, приводящих к понятию производной, можно отнести такие задачи, как проверка прямолинейного движения твердого тела, движение предмета, брошенного вертикально вверх, или движение поршня в цилиндре двигателя. Рассматривая такие движения, игнорируем конкретные размеры и форму тела и представляем его как движущуюся материальную точку.

Ключевые слова: Сложение функции, сложение аргумента, конечный предел, производная, дифференциал.

The volume of a cube is a function of its side length, $V = x^3$. If the cube is made of metal, as the cube heats up, its side length increases, and its volume also increases. If a cube has side length x and increases by h when heated, then it takes $x + h$ and the volume of the cube is $(x + h)^3$. So, when heated, the volume of the cube increased by $(x + h)^3 - x^3$. This difference is called the increase in volume of the cube, and the number h that indicates how much the length of the side of the cube has increased is called the increase in the length of the side of the cube [1-9]. Generally speaking, this gain term is inappropriate because (for example, when the cube cools) the side length of the cube may decrease, in which case the gain will be negative. Therefore, it would be better to call it a change, not an increase, but we will not deviate from the traditional term. Thus, the new value of the quantity x is equal to $x + \Delta x$, that is, it is equal to the sum of its initial value x and Δx. If $y = f(x)$ is a function, and the argument x takes an increment of Δx, then the value of the function also changes, as a result, y gets an increment of Δy. To calculate this gain:

a) Finding the value of the function $y = f(x)$ at the initial value of the argument;

b) Finding the new value of the argument $x + \Delta x$;

c) Finding the new value of the function $f(x + \Delta x)$;

d) It is necessary to subtract the initial value of the function from the new value, that is, find the difference $f(x + \Delta x) - f(x)$;

So,

$$\Delta y = f(x + \Delta x) - f(x).$$

If the 4 th value of the x argument is incremented by 0.1, we find the increment of the function $y = x^2$. At $x = 4$, the value of the function after receiving an increment equal to 16, the value of the argument was 4,1, the new value of the function was equal to 16,81. So the gain of the function is equal to 0,81. If the function $y = f(x)$ grows on the section $[a, b]$, then the signs of Δy and Δx are the same in this section. As the x argument increases, y also increases, and as the x argument decreases, y also decreases [10-21]. If the function $y = f(x)$ decreases in this section, the signs of Δy and Δx will be opposite at any point of it.

The function $y = f(x)$ defined in an interval has a certain value at each value of the argument x in this interval. Let the argument x take any (positive or negative) product of Δx. Then the function y takes an increment Δy. Thus: at the x value of the argument we have $y = f(x)$, at the $x + \Delta x$ value of the argument we have $y + \Delta y = f(x + \Delta x)$. We find the product of the function Δy: $\Delta y = f(x + \Delta x) - f(x)$. We construct the ratio of the increment of the function to the increment of the argument. We construct the ratio of the increment of the function to the increment of the argument:

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

We find the limit of this ratio at $\Delta x \to 0$. If this limit exists, it is called the derivative of the given function $f(x)$ and is denoted by $f'(x)$. So, by definition,

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x},$$

or

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

So, the given function $y = f(x)$ is the derivative of the function with respect to the argument x, and the sum of the argument Δx arbitrarily tends to zero. In general, for each value of x, we emphasize that the derivative $f'(x)$ has a certain value, that is, the derivative is also a function of x.

Let a point M move along a straight line λ at time t by a distance $s = f(t)$. Let $s_0 = f(0)$ be the position of the material point M on the straight line at the initial time $t = 0$. What is its instantaneous speed at arbitrary time t_0 ?

To answer these questions, $\Delta s = |M_0M|$ we determine the path: $\Delta s = f(t_0 + \Delta t) - f(t_0)$. Then we divide by Δt to find the average speed achieved during the time interval $[t_0; t_0 + \Delta t]$:

$$V_{o\prime r} = \frac{\Delta s}{\Delta t} = \frac{f(t_0 + \Delta t) - f(t_0)}{\Delta t}.$$

The smaller the quantity Δt, the closer it really is to the instantaneous speed. Now if we set Δt to zero, then $V_{o\prime r}$ also determines the so-called instantaneous speed at time t_0:

$$V(t_0) = \lim_{\Delta t \to 0} V_{o\prime r} = \lim_{\Delta t \to 0} \frac{f(t_0 + \Delta t) - f(t_0)}{\Delta t}.$$

Thus, $V(t_0) = f'(t_0)$.

Example. Point M is moving along a straight line with law $s(t) = 2\, sin3\pi t$. Find its instantaneous speed in 10 seconds. When will its instantaneous velocity be zero?

Solving. This is a longitudinal oscillating motion. The derivative of the traveled path function $s'(t) = 6\pi cos3\pi t$ is the instantaneous speed of the given point at an arbitrary time t. In that case:

$$V(10) = s'(t_0) = 6\pi cos30\pi = 6\pi.$$

Now we find when its instantaneous speed becomes zero:

$$V(t) = 0 \leftrightarrow \ 6\pi cos3\pi t = 0 \leftrightarrow t_n = \frac{1+2n}{6}, n \in Z.$$

Answer: $V(10) = 6\pi\frac{m}{s},\ T = \left\{t_n = \frac{1+2n}{6}, n \in Z\right\}$.

It is known that if the function f has a derivative at the point x_0, then according to the definition of the derivative:

$$\lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x} = f\,'(x_0).$$

If Δx is a very small quantity, then the quantity $\frac{\Delta f(x_0)}{\Delta x}$ differs from $f\,'(x_0)$ by some infinitesimal amount $\alpha(\Delta x)$:

$$\frac{\Delta f(x_0)}{\Delta x} = f\,'(x_0) + \ \alpha(\Delta x), \qquad \lim_{\Delta x \to 0} \alpha(\Delta x) = 0.$$

Therefore, the approximate equality $\Delta f(x_0) \approx f\,'(x_0)\Delta x$ is formed here. This is an error of approximation

$$|\Delta f(x_0) - f\,'(x_0)\Delta x| \le |\Delta x| max\,|\alpha(\Delta x)|, \qquad (\Delta x\ \to\ 0)$$

satisfies the inequality. So, if we replace the product $\Delta f(x_0)$ by $f\ (x_0 + \Delta x) - f(x_0)$, this approximate equality

$$f\ (x_0 + \Delta x) \approx f(x_0) + f\,'(x_0)\Delta x$$

looks like. At least the value of the function f at the point $x = x_0 + \Delta x$

$$|\Delta x| max\,|\alpha(\Delta x)|$$

This formula is used to determine the error. For example, for the function $f(x) = x^n$, $(x_0 + \Delta x)^n \approx x_0^n + nx_0^{n-1}\Delta x$. In particular: $(x_0 + \Delta x)^2 \approx x_0^2 + 2x_0\ \Delta x$; $(x_0 + \Delta x)^3 \approx x_0^3 + 3x_0^2\Delta x$, $(1 + \Delta x)^n \approx 1 + n\Delta x$.

Example. 1) $(5{,}012)^2 \approx 5^2 + 2 * 5 * 0{,}012 = 25{,}12$.

2) $(1{,}02)^{10} \approx 1 + 10 * 0{,}02 = 1{,}2$.

These formulas are valid around a sufficiently small point $x = x_0$. For example, at sufficiently small values of Δx when $x_0 = 1$:

$$\ln(1 + \Delta x) \approx \Delta x, \quad \sqrt[n]{1 + \Delta x} \approx 1 + \frac{\Delta x}{n}.$$

Let the function $y = f(x)$ be defined in some interval $[a, b]$ and be invertible in it. Again, this function is differentiable in this interval and its derivative does not become zero anywhere. Given these conditions, the function has the following properties:

- $y = f(x)$ the function is continuous in the interval $[a, b]$;

- $f'(x)$ derivative does not change its sign in the interval $[a, b]$, so it is either decreasing or increasing in this interval;

- The values of the function $f(x)$ completely cover an interval $[c, d]$ once, i.e., any arbitrary for points $x_1 \in [a, b]$ and $x_2 \in [a, b]$ from domain $[c, d]$;

The values of the function $f(x)$ completely cover an interval $[c, d]$ once, that is, the derivative of the domain $[c, d]$ for any arbitrary points $x_1 \in [a, b]$ and $x_2 \in [a, b]$ is non-zero, then it is invertible and the derivative of the inverse function $x = f^{-1}(y)$ at the point $y_0 = f(x_0)$ defined as

$$(f^{-1}(y))'_{y=y_0} = \frac{1}{f'(x_0)}.$$

Proof. Based on the property of the function stated in the condition of the theorem, $f(x)$ ensures that the function is either increasing or decreasing in the interval $[a, b]$. So, the function $f(x)$ is definitely invertible in this interval [22-29]. For clarity, let $f(x)$ be increasing in this interval. Now if we make addition $\Delta x = x - x_0$ at arbitrary point $x_0 \in [a, b]$, since $x = f^{-1}(y)$, $x = f^{-1}(y_0)$ $\Delta x = f^{-1}(y) - f^{-1}(y_0)$ or we can write $\Delta x = \Delta f^{-1}(y_0)$. As well as

$$\Delta y = y - y_0 \leftrightarrow \Delta y = f(x) - f(x_0)$$

such a notation shows that the inequality condition $\Delta y \neq 0$ is necessarily fulfilled when $x = x_0$ and $x \in [a, b]$ based on the increasing (decreasing) form of the function f. So this ratio has meaning [30-34]:

$$\frac{\Delta x}{\Delta y} = \frac{1}{\frac{\Delta y}{\Delta x}}.$$

The product of the function f with the derivative at the point x satisfies the condition $\Delta f(x_0) \to 0$ when $\Delta x \to 0$, i.e. the condition $\Delta y \to 0$. So, using them, we can construct this equation:

$$\lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y} = \frac{1}{\lim_{\Delta x \to 0} \frac{\Delta f(x_0)}{\Delta x}}.$$

Since the right-hand denominator has a limit and is non-zero it follows that

$$\left(f^{-1}(y)\right)' = \frac{1}{f'(x_0)}.$$

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