MURAKKAB TUZILISHDAGI ARALASH MAKSIMUMLI DIFFERENSIAL TENGLAMALAR SISTEMALARI UCHUN CHEGARAVIY SHART

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Annotatsiya: Murakkab tuzilishdagi aralash maksimumli differensial tenglamalar sistemalari uchun chegaraviy shart berilgan bo'lib, bu shart asosida differensial tenglamaning yagona yechimga ega bo'lishini ketma-ket yaqinlashish, itaratsiya metodlarini qo'llagan holda isbotlab beramiz.

Kalit so'zlar: Murakkab tuzilishdagi aralash maksimumli differensial tenglamalar, yagona yechim, chegaraviy shart, ketma-ket yaqinlashish metodi, itaratsiya metodi

Ushbu mavzuda chiziqli bo'lmagan tenglamalar sistemasi ko'rib chiqiladi

$$x'(t) = F(t, x(t), \max\{x(r) \mid r \in [t \mid \sigma(t, x(t))]\}), t \in [0;T]$$

$$0 \le \sigma(t,x) \le t \qquad T^2 \equiv \left[\overset{*}{t};T\right] \overset{*}{t} > 0 \qquad (1)$$

Chegaraviylik sharti

$$\begin{cases} x(0) = \varphi_0 \\ x(T) = \varphi_T \end{cases} \qquad (2) \text{ va } (3)$$

Bu yerda $x \in X \subset R^n$ o'suvchi vektori , X chegaralangan yopiq to'plam, $\sigma(t, x(t)$ - kerakli $x(t)$ funksiyaga qarashli $0 \le \sigma(t,x) \le T$ ning $T^1 \equiv \left[0;\overset{*}{t}\right]$ va

$t \le \sigma(t,x) \le T$ ning birinchi qismida $T^2 \equiv \left[t; \overset{*}{T}\right]$ ning ikkinchi qismida $\overset{*}{t} > 0$ shart mavjudligini ko'rib chiqamiz.

Lemma 1. Quyidagilar o'rinli bo'lsin

1. $0 \le \sigma(t,x) \le t$ da $t \in T^1$ (4)

2. $F(t,x,y) \in C\left(T^1 \times X \times X\right) \cap Bnd(M_1) \cap \left(L_{1\backslash x,y}\right)$ (5)

3. $\sigma\left(t,x\right) \in C\left(T^1 \times X\right) \cap Lip\left(L_{2x}\right)$ (6)

$C\left(T^1 \times X\right)$ sinfda (1) boshlang'ich shartga ko'ra (2) tenglama yagona yechimga ega bo'ladi.

Isbot. T^1 segmentda biz ketma-ket yaqinlashishlar yordamida (2) shartga ko'ra (1) tenglama yechimini quydagicha tuzamiz

$$\left\{\begin{aligned} & x_0(t) = \varphi_0 \qquad\qquad t \in T^1 \\ & x_{m+1}(t) = \varphi_0 + \int_0^t F\left(\theta, x_m(\theta), \max\left\{x_m(r) \mid r \in \left[\sigma(\theta, x_m(\theta)); \theta\right]\right\}\right) d\theta \end{aligned}\right\}$$

(7)

Iteratsiya jarayonining farqini (7) orqali baholaymiz.

(5) va (7) ga ko'ra, $x_1(t) - x_0(t)$ birinchi farq uchun quyidagi baholash o'rinli

$$\left\|x_1(t) - x_0(t)\right\| \le M_1 t, \qquad t \in T^{(1)} \tag{8}$$

(5) ga ko'ra $x_2(t) - x_1(t)$ farq quydagicha bo'ladi

$$\left\|x_2(t) - x_1(t)\right\| \le L_1 \int_0^t (\left\|x_1(\theta) - x_0(\theta)\right\| +$$

$$+\left\|\max\left\{x_1(r) \mid r \in \left[\theta \mid \sigma(\theta; x_1(\theta))\right]\right\}\right\| - \left\|\max\left\{x_0(r) \mid r \in \left[\theta \mid \sigma(\theta \mid x_0(\theta))\right]\right\}\right\|) d\theta$$

(9)

(9) ning o'ng tomonidagi ikkinchi qismini quyidagicha yozamiz

$$\max\left\{x_1(r) \mid r \in \left[\theta \mid \sigma(\theta; x_1(\theta))\right]\right\} - \max\left\{x_0(r) \mid r \in \left[\theta \mid \sigma(\theta; x_0(\theta))\right]\right\} =$$

$$\max\left\{x_1(r) \mid r \in \left[\theta \mid \sigma(\theta; x_1(\theta))\right]\right\} - \max\left\{x_0(r) \mid r \in \left[\theta \mid \sigma(\theta; x_0(\theta))\right]\right\} +$$

$$+\max\left\{x_0(r) \mid r \in \left[\theta \mid \sigma(\theta; x_0(\theta))\right]\right\} - \max\left\{x_0(r) \mid r \in \left[\theta \mid \sigma(\theta; x_0(\theta))\right]\right\}$$

(10)

(4) va (8) ni hisobga olgan holda (10) tenglikning birinchi farqi uchun quyidagi baholash o'rinli

$$\left\|\max\left\{x_1(r) \mid r\in\left[\theta \mid \sigma(\theta;x_1(\theta))\right]\right\}-\max\left\{x_0(r) \mid r\in\left[\theta \mid \sigma(\theta;x_0(\theta))\right]\right\}\right\|\le$$

$$\le\left\|\max\left\{(x_1(r)-x_0(r)) \mid r\in\left[\theta \mid \sigma(\theta;x_1(\theta))\right]\right\}\right\|\le M_1 t,\quad t\in T^1$$

(7) ga ko'ra, (10) tenglikning ikkinchi farqi uchun quyidagi bahoni olamiz

$$\left\|\max\left\{x_1(r) \mid r\in\left[\theta \mid \sigma(\theta;x_1(\theta))\right]\right\}-\max\left\{x_0(r) \mid r\in\left[\theta \mid \sigma(\theta;x_0(\theta))\right]\right\}\right\|=0$$

U holda (9) tengsizlik quydagicha

$$\left\|x_2(t)-x_1(t)\right\|\le L_1\int_0^t 2M_1\theta d\theta=2L_1M_1\frac{t^2}{2},\quad t\in T^1 \tag{11}$$

Endi (6) va (11) ga ko'ra $x_3(t)-x_2(t)$ farqga qaraylik

$$\left\|x_3(t)-x_2(t)\right\|\le L_1\int_0^t (\left\|x_2(\theta)-x_1(\theta)\right\|+$$

$$\max\left\{x_1(r) \mid r\in\left[\theta \mid \sigma(\theta;x_1(\theta))\right]\right\}-\max\left\{x_0(r) \mid r\in\left[\theta \mid \sigma(\theta;x_0(\theta))\right]\right\}+$$

$$+\max\left\{x_0(r) \mid r\in\left[\theta \mid \sigma(\theta;x_0(\theta))\right]\right\}-\max\left\{x_0(r) \mid r\in\left[\theta \mid \sigma(\theta;x_0(\theta))\right]\right\}d\theta\le$$

$$\le L_1\int_0^t \left(2+L_2M_1\right)\left\|x_2(\theta)-x_1(\theta)\right\|d\theta\le$$

$$\le 2L_1M_1\left[L_1\left(2+L_2M_1\right)\right]\frac{t^3}{3!}\quad t\in T^1$$

Jarayonni xuddi shunday davom ettirsak to'liq $x_{m+1}(t)-x_m(t)$ farq uchun matematik induksiyani quydagicha olamiz

$$\left\|x_{m+1}(t)-x_m(t)\right\|\le 2L_1M_1\left[L_1\left(2+L_2M_1\right)\right]^{m-1}\frac{t^{m+1}}{(m+1)!}\qquad t\in T^1 \tag{12}$$

(12) dan kelib chiqadiki, $\left\{x_m(t)\right\}$ ketma-ketlik t bo'yicha T^1 segmentda tekis yaqinlashadi. Shuning uchun (1) tenglamaning sistemasi (2) boshlang'ich shart bilan T^1 segmentda $x(t)$ yechimga yaqinlashadi, bunda $x\in C^1(T^1;X)$.

Endi ushbu yechimning yagona ekanligini ko'rsataylik. $C^1(T^1;X)$ sinfidagi (1) sistema ham xuddi shunday (2) boshlang'ich shartga ega bo'lgan boshqa $z(t)$ yechimga ega bo'lsin.

$z(t)$ ni va $x_0(t), x_1(t), x_2(t), ...$ ketma-ketliklarni yaqinlashishlarini solishtiramiz.

(5) ga asosan quydagicha baholaymiz

$$\|z(t)-x_1(t)\| \le L_1\int_0^t \|z(\theta)-x_0(\theta)\| +$$

$$+\left\|\max\left\{z(r)\,|\, r\in\left[\theta\,|\,\sigma(\theta;z(\theta))\right]\right\}-\max\left\{x_0(r)\,|\, r\in\left[\theta\,|\,\sigma(\theta;x_0(\theta))\right]\right\}\right\| \le$$

$$\le 2L_1M_1\int_0^t \theta d\theta = 2L_1M_1\frac{t^2}{2!}$$

(6) va keyingi farq uchun quyidagi tengsizlikni olamiz

$$\|z(t)-x_2(t)\| \le L_1\int_0^t \|z(\theta)-x_1(\theta)\| +$$

$$+\left\|\max\left\{z(r)\,|\, r\in\left[\theta\,|\,\sigma(\theta;z(\theta))\right]\right\}-\max\left\{x_1(r)\,|\, r\in\left[\theta\,|\,\sigma(\theta;x_1(\theta))\right]\right\}\right\| +$$

$$+\max\left\{x_1(r)\,|\, r\in\left[\theta\,|\,\sigma(\theta;x_1(\theta))\right]\right\}-\max\left\{x_1(r)\,|\, r\in\left[\theta\,|\,\sigma(\theta;x_1(\theta))\right]\right\} d\theta \le$$

$$\le L_1\int_0^t \left(2+L_2M_1\right)\|z(\theta)-x_1(\theta)\| d\theta \le$$

$$\le 2L_1M_1\int_0^t \left(2+L_2M_1\right)\frac{t^3}{3!}, \qquad t\in T^1$$

Xuddi shunday

$$\|z(t)-x_3(t)\| \le 2L_1M_1\left[L_1\left(2+L_2M_1\right)\right]^4\frac{t^4}{4!}$$

Va hokozo

Ushbu bahodan ko'rishimiz mumkinki $m\to\infty$ da $\|z(t)-x_m(t)\|\to\infty$

T^1 segmentda t bo'yicha tekis yaqinlashadi.

Bundan kelib chiqqan holda (1) tenglama yechimi (2) boshlang'ich shart bilan $\{x_m(t)\}$ ketma-ketlikka ko'ra $C^1\left(T^1;X\right)$ sinfda yagona yechimga ega

Lemma 2. Quyidagilar o'rinli bo'lsin

1. $t \le \sigma(t,x) \le \overset{*}{t} \quad da \qquad t \in T^2$ (13)

2. $F(t,x,y) \in C\left(T^2 \times X \times X\right) \cap Bnd(M_1) \cap (L_{1|x,y})$ (14)

3. $\sigma(t,x) \in C\left(T^2 \times X\right) \cap Lip(L_{2|x})$ (15)

$C^1\left(T^2;X\right)$ ushbu sinfda (3) shartga ko'ra, (1) tenglama yagona yechimg ega.

Isbot. $C^1\left(T^2;X\right)$ sinfda (1), (3) masala funksional-integral tenglamalar sistemasiga ekvivalent.

$$x(t) = \varphi_T - \int_0^t F\left(\theta, x(\theta), \max\left\{x(r) \mid r \in \left[\theta; \sigma\left(\theta, x(\theta)\right)\right]\right\}\right) d\theta$$

Ushbu masala uchun biz iterativ jarayonni quyidagi ko'rinishda yozamiz

$$\begin{cases} x_0(t) = \varphi_T, \quad t \in T^2 \\ x_0(t) = \varphi_T + \int_0^t F\left(\theta, x_m(\theta), \max\left\{x_m(r) \mid r \in \left[\theta; \sigma\left(\theta, x_m(\theta)\right)\right]\right\}\right) d\theta, \ t \in T^2 \end{cases} \tag{16}$$

(13) , (14) dan (16) da $x_1(t) - x_0(t)$ birinchi farq uchun quyidagi bahoni olamiz

$$\left\| x_1(t) - x_0(t) \right\| \le \int_t^T M_2 d\theta = M_2\left(T - t\right), \qquad t \in T^2 \tag{17}$$

(14) va (17) ga binoan $x_2(t) - x_1(t)$ ikkinchi farq uchun quyidagicha baho o'rinli

$$\left\| x_2(t) - x_1(t) \right\| \le \int_0^T L_1 \left\| x_1(\theta) - x_0(\theta) \right\| +$$

$$+ \left\| \max\left\{ x_1(r) \mid r \in [\theta \mid \sigma(\theta; x_1(\theta))] \right\} - \max\left\{ x_0(r) \mid r \in [\theta \mid \sigma(\theta; x_0(\theta))] \right\} \right\| d\theta \le$$

$$\le \int_0^T L_1 \left\| x_1(\theta) - x_0(\theta) \right\| + \left\| \max\left\{ (x_1(\theta) - x_0(\theta)) \mid r \in [\theta \mid \sigma(\theta; x_1(\theta))] \right\} \right\| d\theta \le$$

$$\le 2L_1 M_2 \frac{(T-t)^2}{2!}, \qquad t \in T^2$$

Oxirgi tengsizlikni (15) shart hisobiga uchinchi farq uchun quyidagi bahoni olamiz

$$\left\| x_3(t) - x_2(t) \right\| \le \int_t^T L_1 \left\| x_2(\theta) - x_1(\theta) \right\| +$$
$$+ \left\| \max\left\{ (x_2(\theta) - x_1(\theta)) \mid r \in \left[\theta \mid \sigma(\theta; x_2(\theta)) \right] \right\} \right\| +$$
$$+ M_2 \left[\sigma(\theta; x_2(\theta)) - \sigma(\theta; x_1(\theta)) \right] d\theta \le$$
$$\le L_1 \int_t^T \left(2 + L_2 M_2 \right) \left\| z_2(\theta) - x_1(\theta) \right\| d\theta \le$$
$$\le 2 L_1 M_2 \left[L_2 \left(2 + L_2 M_2 \right) \right] \frac{(T-t)^3}{3!}, \qquad t \in T^2$$

Bu jarayonni davom ettirib, $\forall m \in N$ uchun T^2 segmentda t bo'yicha tekis yaqinlashuvchi $\{x_m(t)\}$ ketma-ketlikni olamiz. T^2 segmentda (1) sistemasi (3) shart bilan $x(t) \in C^1\left(T^2; X\right)$ sohadagi $x(t)$ ga tekis kuchli yaqinlashadi. 1-Lemmadan sistemaning yechimi yagonaligi aniq isbotlanadi. Bunday ko'rish mumkinki, ya'ni $T^1 \equiv \left[0; \overset{*}{t}\right]$ kesmada $\sigma\left(t, x(t)\right) \ge t$ bo'lganda va $T^2 \equiv \left[\overset{*}{t}; T\right]$ kesmada $\sigma\left(t, x(t)\right) \le t$ bo'lganda shunday $\overset{*}{t} > 0$ mavjud bo'lar ekan.

Lemma 3. Quyidagilar o'rinli bo'lsin

1. $0 \le t \le \sigma\left(t, x\right)$ da $t \in T^1$
2. (14) va (15) shartlari bajariladi.

(1) tenglamaning $C^1\left(T^2; X\right)$ sinfda (3) shartga ko'ra yechimi yagonadir.

Lemmaning isboti 2-lemmaning isbotiga o'xshaydi. Bu ikki lemmaning o'rinli ekanidan quyidagicha xulosaga kelamiz.

Teorema. 1 va 2 lemmalar shartlari bajarilsin. Unda $\left[0; T\right]$ oraliqdagi (1) ,(3) chegaraviy masala yagona yechimga ega bo'ladi.

$$x(t) = \begin{cases} \varphi_0 + \int_0^t F\left(\theta, x_m(\theta), \max\left\{ x_m(r) \mid r \in \left[\theta; \sigma\left(\theta, x_m(\theta)\right)\right] \right\}\right) d\theta, & t \in T^1 \\ \varphi_T - \int_t^T F\left(\theta, x_m(\theta), \max\left\{ x_m(r) \mid r \in \left[\theta; \sigma\left(\theta, x_m(\theta)\right)\right] \right\}\right) d\theta, & t \in T^2 \end{cases}$$

Birinchi turdagi uzilishga ega , aniqrog'i $0 < \overset{**}{t} < T$ mavjud bo'lgan holatda, $T^1 \equiv \left[0; \overset{**}{t}\right]$ da $\sigma\left(t, x(t)\right) \geq t$ va $T^2 \equiv \left[\overset{**}{t}; T\right]$ da $\left[0;T\right]$ $\sigma(t, x(t) \leq t$ mana shularga o'xshashdir.

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