XOSMAS INTEGRALLAR VA ULARNING BA'ZI TATBIQLARI

Abdullayeva Gulasal Abdumo'minovna
Toshkent iqtisodiyot va pedagogika instituti
"Axborot texnologiyalari va aniq fanlar"
kafedrasi matematika fani o'qituvchisi

Annotatsiya: Ushbu maqolada xosmas integrallar va uning turlari, ba’zi tadbiqlari va xosmas integrallarni yechish bo’yicha misollar keltirib o’tilgan.

Kalit so’zlar: xosmas integral, xosmas integrallar turlari,yaqinlashuvchi, uzoqlashuvchi.

Aniq integralni ta`riflashda integrallash oralig`i [*a;b*] ni chekli hamda unda aniqlangan *f(x)* integral osti funksiyasi chegaralangan bo`lishini talab qilgan edik. Bunga sabab qo`yilgan bu shartlardan birortasi bajarilmagan taqdirda integral yig`indi mabjud bo`lmay qolishi mumkinligidir. Ammo, bu shartlar bajarilmagan taqdirda ham integral tushunchasini kiritish mumkin bo`lib, bunday holda uni *xosmas integral* deb ataladi. Bu yerda xosmas integral tushunchasini integrallash oralig`i cheksiz bo`lgan, integrallash oralig`i chekli bo`lib, unda integral osti funksiyasi chegaralanmagan va nihoyat, yuqoridagi ikkala hol ham mavjud bo`lgan hollar uchun alohida kiritamiz

. 1-tur xosmas integral

$y = f(x)$ funksiya $[a,+\infty)$ oraliqda aniqlangan va uzluksiz bo`lsin (1-rasm). $\int_a^b f(x)dx$ integralni qaraymiz.

$[a,+\infty)$ oraliqda $f(x)$ funksiyaning **1-tur xosmas integrali** deb, qu-yidagi

$$\lim_{b\to\infty}\int_a^b f(x)dx$$

limitga aytiladi va $\int_a^{+\infty} f(x)dx$ kabi belgilanadi, ya`ni

$$\int_a^{+\infty} f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx \qquad (1)$$

Agar limit mavjud va chekli bo`lsa, u holda xosmas $\int_a^{+\infty} f(x)dx$ integral **yaqinlashuvchi** deyiladi. Bu limit integralning qiymati sifatida qabul qilinadi.

Agar limit mavjud bo`lmasa yoki xususan cheksiz bo`lsa, xosmas integral **uzoqlashuvchi** deyiladi.

ISSN: 3030-3621

Xuddi shuningdek, 1-tur xosmas integral $(-\infty,b]$ oraliq uchun $\int_{-\infty}^{b} f(x)dx = \lim_{a\to-\infty}\int_{a}^{b} f(x)dx$ kabi aniqlanadi (2-rasm).

Faraz qilaylik, $f(x)$ funksiya $(-\infty;+\infty)$ oraliqda aniqlangan va uzluksiz hamda $c\in(-\infty;+\infty)$ bo`lsin. U holda xosmas integrallar:

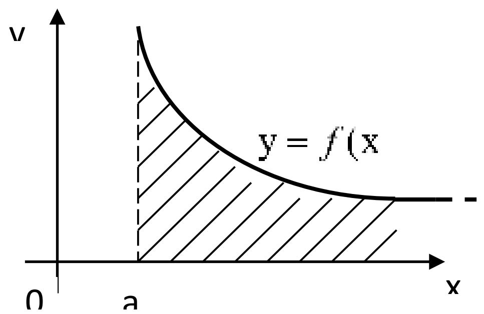
$$\int_{-\infty}^{c} f(x)dx + \int_{c}^{+\infty} f(x)dx$$

yig`indisi $f(x)$ funksiyaning $(-\infty;+\infty)$ oraliqdagi 1-tur xosmas integrali deb ataladi va $\int_{-\infty}^{+\infty} f(x)dx$ kabi belgilanadi.

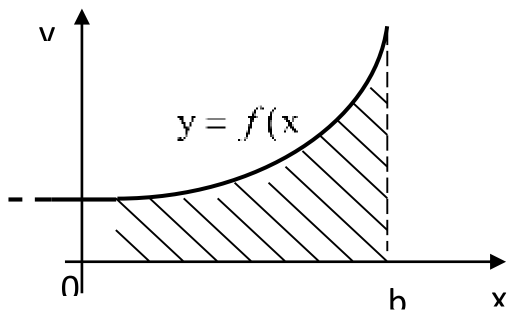
$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^{c} f(x)dx + \int_{c}^{+\infty} f(x)dx \qquad (2)$$

Shunday qilib, (2) yig`indidagi har bir xosmas integral yaqinlashuvchi bo`lsa, xosmas integral ham yaqinlashuvchi bo`ladi. Bu holda (2) yig`indi s nuqtaning tanlanishiga bog`liq bo`lmaydi.

1) $\int_{1}^{+\infty}\frac{dx}{\sqrt{x}} = \lim_{b\to+\infty}\int_{1}^{b}\frac{dx}{\sqrt{x}} = \lim_{b\to+\infty} 2\sqrt{x}\Big|_{1}^{b} = \lim_{b\to+\infty}(2\sqrt{b} - 2\sqrt{1}) = 2\lim_{b\to+\infty}(\sqrt{b} - 1) = +\infty$.



1-rasm



2-rasm

Demak, ushbu integral uzoqlashuvchi ekan.

2) $\int_{-\infty}^{+\infty}\frac{dx}{1+x^2} = \lim_{\substack{a\to-\infty\\ b\to+\infty}}\int_{a}^{b}\frac{dx}{1+x^2} = \lim_{\substack{a\to-\infty\\ b\to+\infty}} \operatorname{arctg}x\Big|_{a}^{b} = \lim_{\substack{a\to-\infty\\ b\to+\infty}}(\operatorname{arctg}b - \operatorname{arctg}a) =$

$= \frac{\pi}{2} - (-\frac{\pi}{2}) = \pi$

Demak, xosmas integral yaqinlashuvchi ekan.

2. 2-tur xosmas integral

$f(x)$ funksiya $[a,b)$ oraliqda aniqlangan va uzluksiz bo`lib, $x = b$ nuqta atrofida chegaralanmagan bo`lsin (3-rasm). U holda

$$\lim_{\varepsilon\to 0}\int_a^{b-\varepsilon} f(x)dx$$

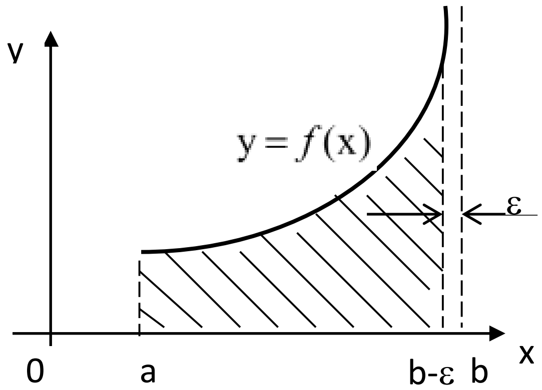
limitga [a,b) oraliqda $f(x)$ funksiyasining **2-tur xosmas integrali** deyiladi:

$$\int_a^b f(x)dx = \lim_{\varepsilon\to 0}\int_a^{b-\varepsilon} f(x)dx \qquad (3)$$

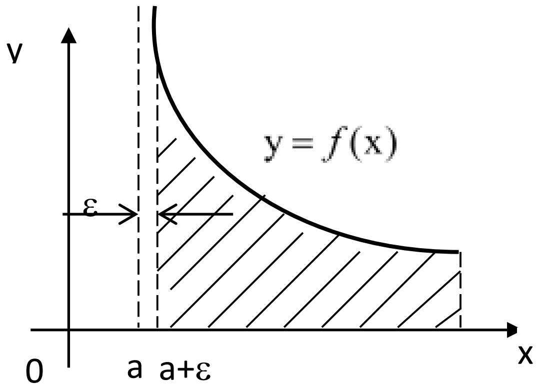
Agar (3) limit mavjud va chekli bo`lsa, xosmas integral yaqinlashuvchi deyiladi. Agar limit mavjud bo`lmasa yoki cheksizga teng bo`lsa, xosmas integral uzoqlashuvchi deb ataladi. (a,b] oraliqda aniqlangan, uzluksiz va x = a nuqta atrofida chegaralanmagan funksiya uchun xosmas integral xuddi shuningdek aniqlanadi (4-rasm):

$$\int_a^b f(x)dx = \lim_{\varepsilon\to 0}\int_{a+\varepsilon}^{b} f(x)dx$$

$f(x)$ funksiya [a, b] oraliqning c∈[a,b] nuqtasidan tashqari barcha nuqtalarida aniqlangan va uzluksiz bo`lib, x = c nuqtaning atrofida



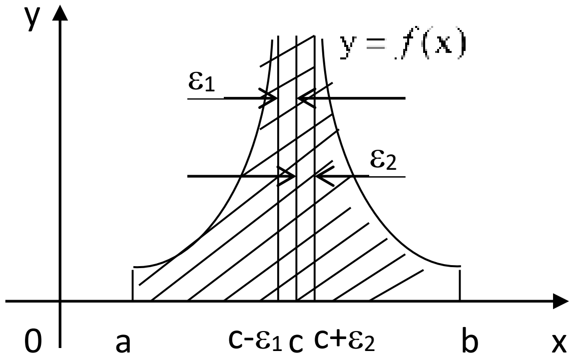
3-rasm



4- rasm

chegaralanmagan bo`lsin (5-rasm). U holda bu funksiyaning [a, b] kesmadagi 2-tur xosmas integrali xosmas integrallarning yig`indisi kabi aniqlanadi:

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \qquad (5)$$



5-rasm

Agar (5) formulaning o`ng tarafidagi har bir xosmas integral yaqinlashuvchi bo`lsa, $f(x)$ funksiyadan [a,b] oraliqda olingan xosmas integral ham yaqinlashuvchi bo`ladi.

Misollar:

1) $\int_0^1 \frac{dx}{\sqrt{1-x}}$ xosmas integralni hisoblang. Integral ostidagi $y = \frac{1}{\sqrt{1-x}}$ funksiya x = 1 nuqtada uzilishga ega. Demak,

$$\int_0^1 \frac{dx}{\sqrt{1-x}} = \lim_{\varepsilon\to+0} \int_0^{1-\varepsilon} \frac{dx}{\sqrt{1-x}} = \lim_{\varepsilon\to+0} \left(-2\sqrt{1-x}\right)\Big|_0^{1-\varepsilon} = -2\lim_{\varepsilon\to+0}(\sqrt{1-1+\varepsilon} - \sqrt{1-0}) =$$

$$= -2\lim_{\varepsilon\to+0}(\sqrt{\varepsilon} - \sqrt{1}) = 2$$

2) $\int_0^2 \frac{dx}{(x-1)^2}$ xosmas integralni hisoblang.

Integral ostidagi $y = \frac{1}{(x-1)^2}$ funksiya x = 1∈[0,2] nuqtada 2-tur uzilishga ega.

Demak,

$$\int_0^2 \frac{dx}{(x-1)^2} = \lim_{\varepsilon_1\to0} \int_0^{1-\varepsilon_1} \frac{dx}{(x-1)^2} + \lim_{\varepsilon_2\to0} \int_{1+\varepsilon_2}^2 \frac{dx}{(x-1)^2} = -\lim_{\varepsilon_1\to0} \frac{1}{x-1}\Big|_0^{1-\varepsilon_1} - \lim_{\varepsilon_2\to0} \frac{1}{x-1}\Big|_{1+\varepsilon_2}^2 =$$

$$= -\lim_{\varepsilon_1\to0}\left(-\frac{1}{\varepsilon_1}+1\right) - \lim_{\varepsilon_2\to0}\left(1-\frac{1}{\varepsilon_2}\right) = \infty$$

Demak, berilgan integral uzoqlashuvchi ekan.

Foydalanilgan adabiyotlar ro'yxati

1. Азларов Т., Мансуров Х. ,Математик анализ,Т.: «Ўқитувчи». 1 т: 1994 й. 315 б.

2. Азларов Т., Мансуров Х. ,Математик анализ,Т.: «Ўқитувчи». 2 т: 1995 й. 336 б.
3. Аюпов Ш.А., Бердиқулов М.А.,Функциялар назарияси ,Т.: “ЎАЖБНТ” маркази, 2004 й. 148 б.
4. Turgunbayev R.,Matematik analiz. 2-qism,T.TDPU, 2008 y.
5. Jo‘raev T. va boshqalar,Oliy matematika asoslari. 2-q.,T.: «O‘zbekiston». 1999
6. Саъдуллаев А. ва бошқ.Математик анализ курсидан мисол ва масалалар тўплами, III қисм. Т.: «Ўзбекистон», 2000 й., 400 б.