$sl(2)$ ALGEBRANING ANTI-ROTA-BAKSTER OPERATORLARI

Azizov Majidxon Ergashxon o'g'li.
Alfraganus universiteti, Toshkent.
azizovmajidkhan@gmail.com.

Rota-Bakster operatorlari dastlab Jan-Karlo Rota va Richard Baksterlarning ishlariga foydalanilgan bo'lib, hozirgi kunda matematikaning bir qator sohalariga tarqalib bormoqda. Ushbu operatorlar avvalllari kombinatorika va ehtimollar nazariyasida ko'plab foydalanilgan bo'lsa, so'nggi paytlarda algebrada ham keng qo'llanilmoqda.

Glen Bakster, 1960-yilda Rota-Bakster operatorlari yordamida ehtimollik taqsimotlariga oid ma'lum bir xususiyatni qanoatlantiruvchi quyidagi maxsus Bakster tenglamasini kiritdi [3]:

$$2\left(a\left(aT\right)\right)T=\left(a^2b\right)T+\left(aT\right)^2$$

bu erda, A algebra va $b\in A$ undagi element, $\forall a\in A,\, T\in End\left(A\right)$.

Pre-Li algebrasini tavsiflashda, Li algebrasidagi O-operatorlaridan foydalanilgani kabi [2], anti-pre-Li algebrasini tavsiflash uchun anti-O-operatorlari muxim ahamiyat kasb etadi [4].

Ushbu ishda $sl(2,\mathbb{C})$ Li algebrasida anti-pre-Li algebra strukturasini qurishga imkon beruvchi anti-Rota-Bakster operatorlarini tavsifini keltiramiz.

Umumiy holatda Kupershmidt tomonidan anti-O-operatorlarni qurish strukturasi keltirilgan [1]. Unga ko'ra, anti-Rota-Bakster operatoriga quyidagicha ta'rif beriladi:

Ta'rif 1. [4] Aytaylik $(g,[-,-])$ Li algebrasi berilgan bo'lsin. Agar $R:g\to g$ chiziqli operator uchun

$$[R(x),\ R(y)] = R([R(y),\ x] + [y,\ R(x)]), \forall x,\ y\in g \qquad (1)$$

ayniyat o'rinli bo'lsa, u holda R **anti-Rota-Bakster operatori** deyiladi.

Ta'rif 2. [4] Aytaylik $(g,[-,-])$ Li algebrasi berilgan bo'lsin. Agar $R:g\to g$ anti-Rota-Bakster operatori

$$\left[\left[R(x),R(y)\right],z\right] + \left[\left[R(y),R(z)\right],x\right]+\left[\left[R(z),R(x)\right],y\right]=0,\ \forall x,y,z\in g \qquad (2)$$

shartni qanoatlantirsa, u holda R **kuchli anti-Rota-Bakster operatori** deyiladi.

Kompleks sonlar maydonida ushbu 3-o'lchamli $sl(2,\mathbb{C})=\{x,y,z\}$ maxsus Li algebrasini qaraylik :

$$x=\begin{pmatrix}0&1\\0&0\end{pmatrix},\ y=\begin{pmatrix}0&0\\1&0\end{pmatrix},\ z=\begin{pmatrix}1&0\\0&-1\end{pmatrix},$$

u holda quyidagi ko‘paytirish jadvaliga ega bo‘lamiz:

$$[x,y]=z,\ [x,z]=2x,\ [y,z]=-2y. \qquad (3)$$

Endi ushbu algebra uchun anti-Rota-Bakster operotorlarini topamiz. R matritsani quyidagi ko‘rinishda olaylik:

$$R=\begin{pmatrix} a & b & c \\ d & g & h \\ k & l & m \end{pmatrix}\in M_3(\mathbb{C}).$$

Ushbu R matritsa va $sl(2,\mathbb{C})$ algebra uchun (1) va (2) ayniyatlarni qo‘llagan holda, anti-Rota-Bakster va mos ravishda kuchli anti-Rota-Bakster operatorlarini olamiz:

Tasdiq. Kompleks sonlar maydonida ushbu 3-o‘lchamli $sl(2,\mathbb{C})=\{x,y,z\}$ maxsus Li algebrasi uchun quyidagicha anti-Rota-bakster operatorlari mavjud:

$$\begin{pmatrix} 0 & 0 & 0 \\ d & 0 & h \\ 0 & 0 & 0 \end{pmatrix},\ \begin{pmatrix} 0 & 0 & 0 \\ d & 0 & h \\ 2h & 0 & 0 \end{pmatrix},\ \begin{pmatrix} 0 & 0 & c \\ d & 0 & 0 \\ 0 & 2c & m \end{pmatrix},\ \begin{pmatrix} -2m & 0 & 0 \\ d & -2m & h \\ 2h & 0 & m \end{pmatrix},\ \begin{pmatrix} 0 & b & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{zoo}$$

$$\begin{pmatrix} a & 0 & c \\ -\frac{a^3}{4c^2} & 0 & -\frac{a^3}{4c^2} \\ -\frac{a^2}{c} & 0 & -a \end{pmatrix} (\text{bu erda } c\neq 0),\quad \begin{pmatrix} g & 0 & c \\ d & g & h \\ 2h & 2c & -\frac{g}{2}+\frac{2ch}{g} \end{pmatrix} (\text{bu erda } g\neq 0),$$

$$\begin{pmatrix} g & b & c \\ \frac{-4ch+gm+g(g+m)}{b} & g & h \\ 2h & 2c & m \end{pmatrix} (\text{bu erda } b\neq 0).$$

Bularning ichida quyidagi operatorlar

$$\begin{pmatrix} 0 & 0 & 0 \\ d & 0 & h \\ 0 & 0 & 0 \end{pmatrix},\ \begin{pmatrix} 0 & 0 & 0 \\ d & 0 & h \\ 2h & 0 & 0 \end{pmatrix}$$

kuchli anti-Rota-Bakster operatorlaridir.

Foydalanilgan adabiyotlar:

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