ANALYSIS OF SOME APPROXIMATE METHODS USED IN SOLVING 1ST AND 2ND TYPE FREDHOLM INTEGRAL EQUATIONS AND FINDING THEIR CHARACTERISTIC VALUES

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Annotation. In this article, the approximate calculation methods used in finding the solutions of Fredholm integral equations, which are one of the important types of linear integral equations, and determining their characteristic numbers, are studied, analyzed, and related examples are solved.

Keywords. Fredholm integral equation, characteristic value of integral equation, vector function, Bubnova-Galyorkina method, Kelloga method.

АНАЛИЗ НЕКОТОРЫХ ПРИБЛИЖЕННЫХ МЕТОДОВ, ИСПОЛЬЗУЕМЫХ ПРИ РЕШЕНИИ ИНТЕГРАЛЬНЫХ УРАВНЕНИЙ ФРЕДГОЛЬМА 1-ГО И 2-ГО ТИПОВ, И НАХОЖДЕНИЕ ИХ ХАРАКТЕРИСТИЧЕСКИХ ЗНАЧЕНИЙ.

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Аннотация. В данной статье изучаются, анализируются и решаются соответствующие примеры приближенные методы расчета, используемые при нахождении решений интегральных уравнений Фредгольма, которые являются одним из важных типов линейных интегральных уравнений, и определении их характеристических чисел.

Ключевые слова. Интегральное уравнение Фредгольма, характеристическое значение интегрального уравнения, векторная функция, метод Бубновой-Галёркиной, метод Келлоги.

Integral equations are considered one of the important concepts for many areas of mathematics, and have effective applications in areas such as functional analysis, differential equations, theory of integral operators[1-5]. This article explores some approximate solution methods and their interrelationships used in solving Fredholm integral equations of type 1 and 2. One of the methods of approximate solution of type 2 Fredholm integral equations is the Bubnova Galyorkina [1] method. from us

$$\varphi(x)-\lambda\int_a^b K(x,y)\,\varphi(y)dy=f(x) \qquad (1)$$

an approximate solution of the integral equation of the form is required.To use the Bubnova-Galyorkina method, we obtain a sequence of functions $\{U(x)\}$ that are complete and linearly independent in the space of functions integrable by the square of $L_2[a,b]$. Using them, we look for the approximate solution of $\varphi_n(x)$ in the form of the sum below [6-11].

$$\varphi_n(x)=\sum_{k=1}^{n}\alpha_k U_k(x),\quad k=1,2,\dots,n \qquad (2)$$

We determine the unknown α_k coefficients through the following system of linear equations:

$$\big(\varphi_n(x),U_k(x)\big)=\big(f(x),U_k(x)\big)+\lambda\left(\int_a^b K(x,t)\,\varphi_n(t)dt,U_k(x)\right),$$

$$k=1,2,\dots,n$$

It is known that $L_2[a,b]$ is a scalar product in real variable space

$$(f,g)=\int_a^b f(x)g(x)dx$$

determined by the formula. Putting the values of the found unknown α_k, $k=1,2,\dots,n$ coefficients into equation (2), we get the value of the approximate solution $\varphi_n(x)$. If the number λ is not a characteristic value of the integral equation (1), then for sufficiently large n the integral equation (3) has a unique solution. Also, when $n\to\infty$, the approximate solution (2) approaches the exact solution $\varphi(x)$ defined in the space $L_2[a,b]$ of (1). Below we solve the given integral equation using the Bubnova Galyorkina method [12-16].

$$\varphi(x) = \lambda \int_{-1}^{1} xt\,\varphi(t)dt + x \tag{4}$$

Solve: We can obtain the system of Legendre polynomials $P_n(x)$ as a complete and linearly independent system in the interval [-1,1].

$$P_0(x) = 1, \quad P_n(x) = \frac{1}{2^n n!} \frac{d^n(x^2-1)^n}{dx^n}, \qquad n = 1, 2, 3, \dots$$

First, we use the first 3 terms of the Legendre polynomials, the more terms we use, the more accurate the solution.

$$P_0(x) = 1, \ \ P_1(x) = x, \ \ P_2(x) = \frac{3x^2-1}{2}$$

$$\varphi_3(x) = \alpha_1 \cdot 1 + \alpha_2 \cdot x + \alpha_3 \frac{3x^2-1}{2} \tag{5}$$

We put the obtained approximate solution (5) into the integral equation (4) and determine the unknown coefficients α_k by the method of equalization of unknown coefficients [17-21].

$$\alpha_1 \cdot 1 + \alpha_2 \cdot x + \alpha_3 \frac{3x^2-1}{2} = x + \int_{-1}^{1} xt\left[\alpha_1 \cdot 1 + \alpha_2 \cdot t + \alpha_3 \frac{3t^2-1}{2}\right]dt$$

As a result of calculations, we get that

$$\alpha_1 = \alpha_3 = 0, \ \ \alpha_2 = 3$$

As a result, for an approximate solution

$$\varphi(x) \approx 3x$$

attitude will be appropriate

In the next part of our article, we will analyze the use of the "Kelloga" [1] method, which is one of the methods of approximate calculation of eigenvalues and eigenfunctions of integral equations with a symmetric core.

It is known that the eigenvalues of an integral equation have important applications for the theory of integral operators. To find the characteristic values and eigenvector functions of the Fredholm integral equation of the first and second type, we construct the homogeneous Fredholm integral equation corresponding to them. We can approximately calculate the eigenvalues of the given integral equation and their corresponding eigenvector functions [22-28]. We get a function $\omega(x)$ that takes an arbitrary positive value and is continuous in the space $L_2[a, b]$. Using this function and the kernel of integral equations, we construct the following sequence of functions.

$$\begin{cases} \omega_1(x) = \int_a^b K(x,t)\omega(t)dt \\ \omega_2(x) = \int_a^b K(x,t)\omega_1(t)dt \\ \dots\dots\dots \\ \omega_n(x) = \int_a^b K(x,t)\omega_{n-1}(t)dt \\ \dots\dots\dots\dots \end{cases}$$

Using the generated sequence of functions, we generate the following numerical sequence

$$\left\{\frac{\|\omega_{n-1}\|}{\|\omega_n\|}\right\}.$$

A sequence of functions $\varphi_1(x), \varphi_2(x), \dots$ is a system of vector functions forming a mutually orthonormal system corresponding to $\lambda_1 \le \lambda_2 \le \lambda_3 \le \cdots$ eigenvalues of an integral equation with $K(x,t)$ kernels. If our arbitrarily selected function $\omega(x)$ is orthogonal to the found vector functions $\varphi_1(x), \varphi_2(x), \dots, \varphi_{k-1}(x)$ and is not orthogonal to the function $\varphi_k(x)$, then We can determine the characteristic values corresponding to the eigenvector φ_k (x) by the following approximate calculation formulas.

$$\lambda_1 \approx \frac{\|\omega_{n-1}\|}{\|\omega_n\|}$$

$$\lambda_1 \approx \frac{1}{\sqrt[n]{\|\omega_n\|}}$$

Below, we use Kellogg's method to approximate the characteristic values of the integral equation corresponding to the given symmetric core.

Example 2.

$$K(x,t) = xt, \quad 0 \le x,t \le 1$$

Approximate the characteristic values of integral integral equations.

The solution:

Let's take the function ω(x)=x, as a result we will create the following sequence of functions

$$\begin{cases} \omega_1(x) = \int_0^1 xttdt = \frac{x}{3} \\ \omega_2(x) = \int_a^b xt\frac{t}{3}dt = \frac{x}{9} \\ \dots\dots\dots \\ \omega_n(x) = \int_a^b xt\frac{t}{3^{n-1}}dt = \frac{x}{3^n} \\ \dots\dots\dots\dots \end{cases}$$

$$\|\omega_n\| = \sqrt{\int_0^1 \frac{x^2}{3^{2n}}} = \frac{1}{\sqrt{3}3^n}$$

$$\lambda_1 \approx \frac{\|\omega_{n-1}\|}{\|\omega_n\|} = 3$$

In this way, we can approximately calculate the characteristic values.

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